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A Cost-Benefit Analysis of the Covid-19 Disease

Bob Rowthorn and Jan Maciejowski
Economist Control Engineer

University of Cambridge

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Outline

- The problem
- The SIR model
  - Basic model and some variations
- An optimal control formulation
  - Solution: The classical approach and a pragmatic approach
- Parameters for the British case
- The results
  - Value of life? Early start, Test and Trace
- Should the government use these results?
- Conclusion

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The Problem

- Is the cure worse than the disease?
- How to trade off economic damage against loss of life?
- We assume that a vaccine will be available in one year. What to do until then?

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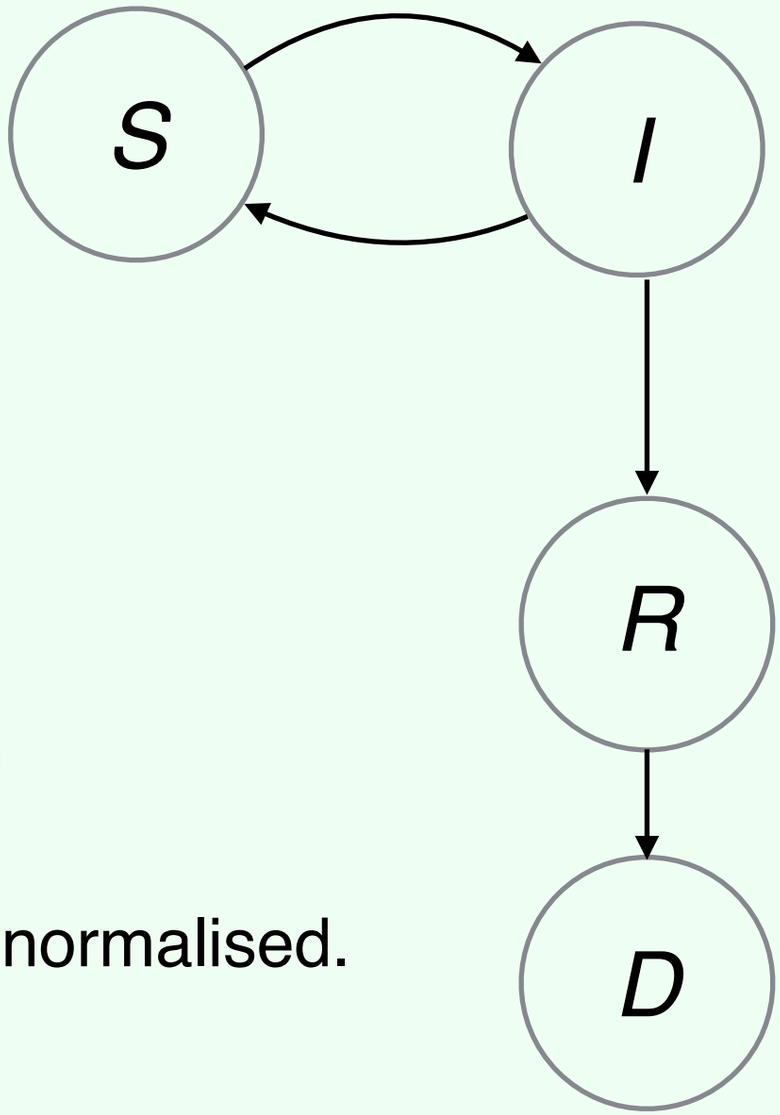
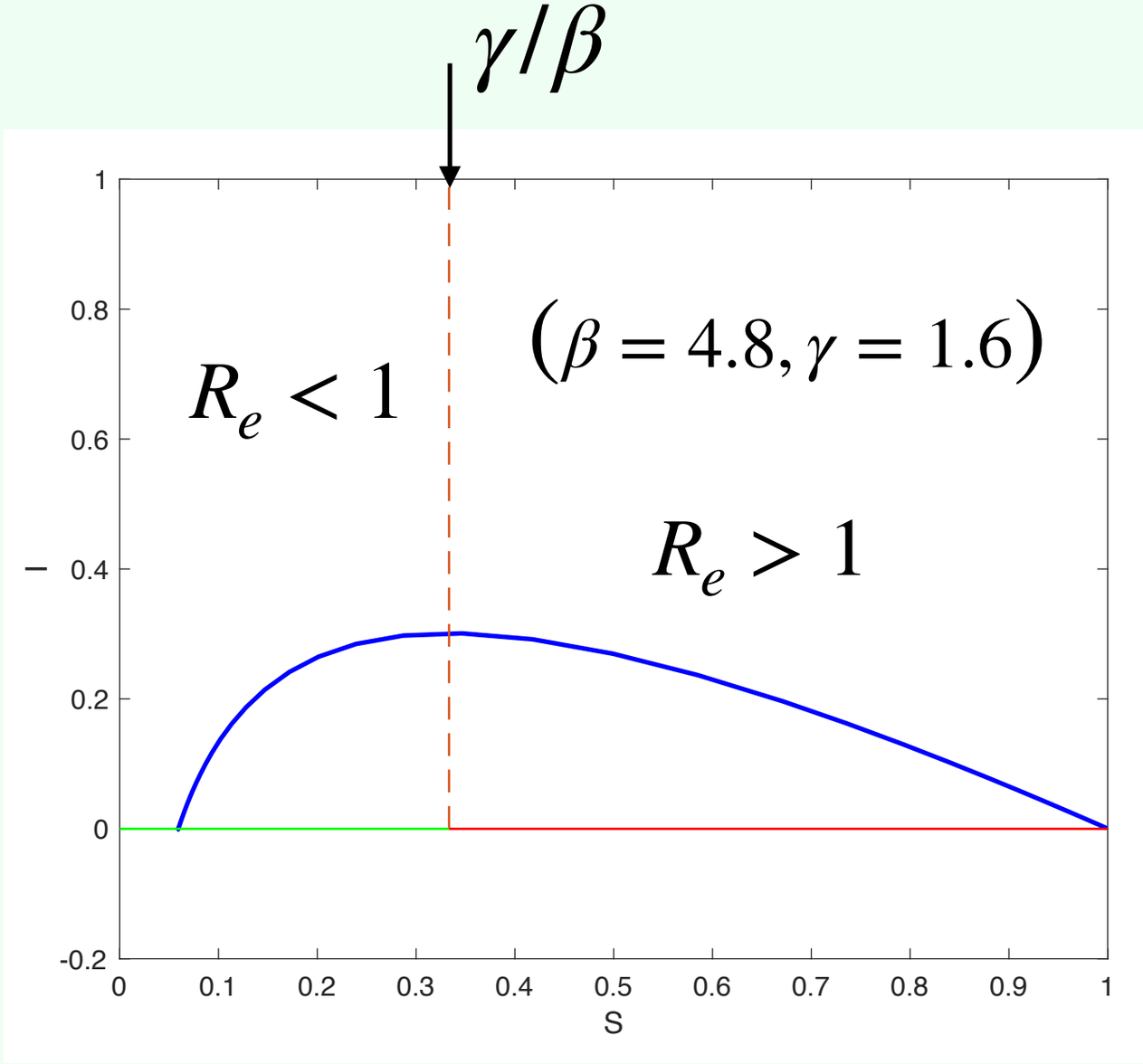
The SIR model

- "SIR" = "Susceptible, Infected, Removed"
- "Removed" = Recovered + Dead
- $\frac{dS}{dt} = -\beta(t)S(t)I(t)$
- $\frac{dI}{dt} = \beta(t)S(t)I(t) - \gamma I(t)$
- $\frac{dR}{dt} = \gamma I(t), \qquad \frac{dD}{dt} = \delta\frac{dR}{dt} = \delta\gamma I(t)$
- $S(t) + I(t) + R(t) = 1$ Total population, normalised.
- Used in epidemiology since 1927.



The SIR model

- $I = 0$ is equilibrium (for any S).
- If $\beta(t) = \beta$ then $(S,0)$ stable if $S < \gamma/\beta$.
- Always stable and $dI/dt < 0$ if $R_0 \triangleq \beta/\gamma < 1$.
- $R_e \triangleq SR_0$
- But $\beta(t)$ is our control variable - not constant!



The SIR model

- More elaborate versions exist (but not used by us)
- *SEIR* - Susceptible, Exposed, Infectious, Removed

  our *I*
- *SIDARTHE* - Susceptible, Infected, Diagnosed, Ailing, Recognized, Threatened, Healed, Extinct

  our *R*
- SIR on random graphs
- SIR decomposed by geography or demography or …

Optimal Control Formulation

- Assume control is

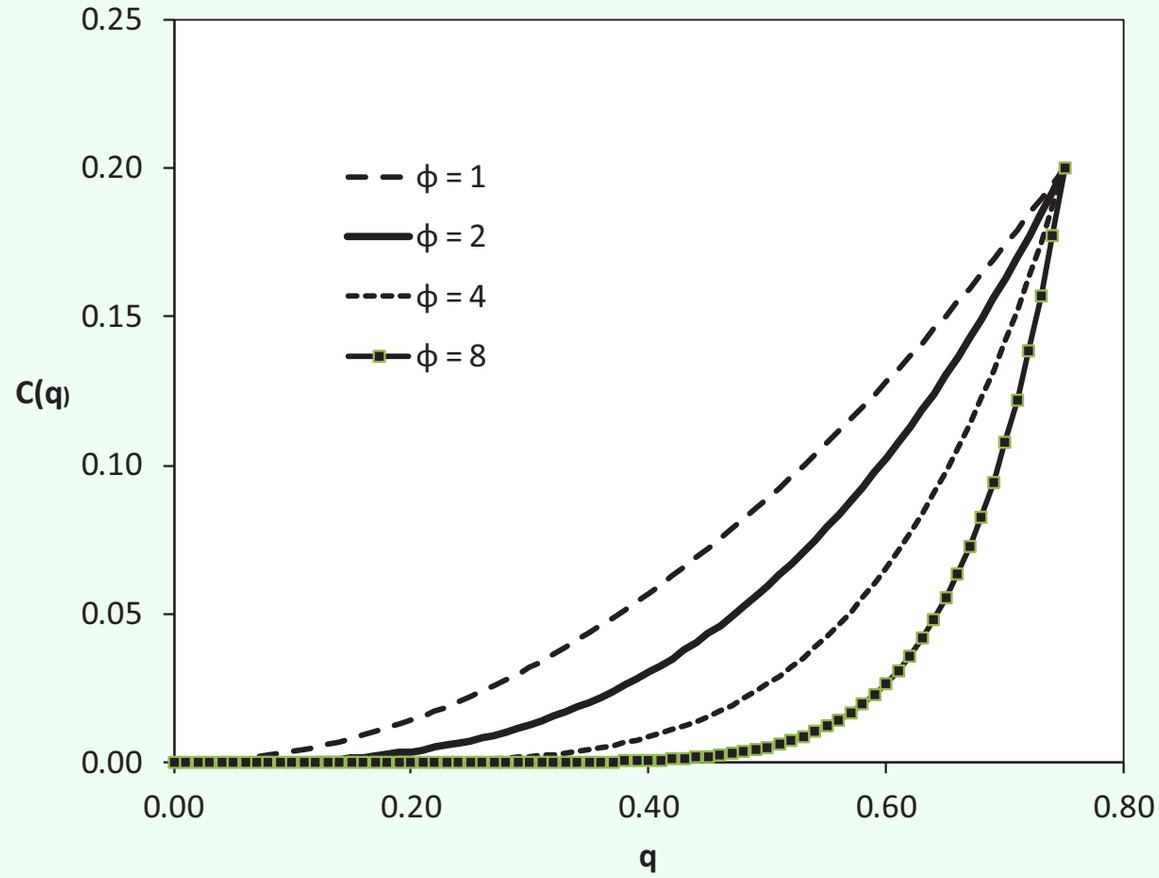
$$\beta(t) = [1 - q(t)]\beta_0, \quad 0 \le q(t) \le q_{max}$$

(no endogenous behaviour)

- Cost of control is

$$C(q) = C_{max}\left(\frac{q}{q_{max}}\right)^{1+\phi}$$

- $q_{max} = 0.75$ seems realistic for the UK



Convex: marginal cost increases as *q* increases.

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Optimal Control Formulation

$$J = \underbrace{\int_0^T e^{-\rho t}\left[\pi_A I(t) + C(q(t))\right] dt + \pi_D \left[D(T) - D(0)\right]}_{E=\textit{Economic}\ \text{cost}}$$

π_A is value of person who is alive and infected

π_D is *additional* value of person who dies

Vaccine and cure become available at $t=T$ at negligible cost

$T = 52$ weeks, $\quad \rho = 0.$

Problem: $\min_{q(t),\ 0\le t\le T} J$ subject to SIR model

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Classical solution by Maximum Principle

Hamiltonian: $H = -\pi_A I - C(q) + \lambda_S \frac{dS}{dt} + \lambda_I \frac{dI}{dt}$

Optimal $q^*(t)$ must maximise H at each t.

This leads to $\frac{d\lambda_S^*}{dt} = -\frac{\partial H}{\partial S}$ and $\frac{d\lambda_I^*}{dt} = -\frac{\partial H}{\partial I}$

almost everywhere with $\lambda_S^*(T) = \lambda_I^*(T) = 0$.

Iterate solutions for (S, I) forwards from $t = 0$
and for (λ_S, λ_I) backwards from $t = T$. Hope for convergence.

Very hard!

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Practical solution by numerical optimisation

- Discretise time into small steps τ (τ = 0.1 week)

- Optimise $q_1, q_2, \ldots, q_{T/\tau}$ subject to dynamics and $0 \leq q_k \leq q_{max}$ and other constraints.

- Nonlinear programming problem — local optima etc.

- More fancy methods exist
  - optimise over interpolating splines
  - multiple shooting, etc
  - Software: ACADO, CasADi, ICLOCS, and others.

- We used the *MPC Toolbox for Matlab,* with *fmincon* as the optimiser — because of easy learning curve.

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Parameters (fixed)

- $t = 0$ is 1 April 2020. UK population is 66.8 million.
- $I(0) = 0.030, \ R(0) = 0.021$ — epidemic started a few weeks earlier.
- $\beta_0 = 4.8, \ \gamma = 1.6 \ (\Rightarrow R_0 = 3$ if $q = 0)$.
  96% not infectious after 2 weeks.
- Death rate $\delta = 0.007 \ (0.7\%)$.
- $C_{max} = 0.2 —$ *per capita* weekly cost of full lockdown is £200, 35% of *per capita* Gross Domestic Product.

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Baseline parameters

- $\pi_A = 2$. Value of average infected person is £2000 per week. This includes cost of treatment (often zero).

- $\pi_D = 2000$. Value of each death is £2 million. (This is the value used by the UK Treasury for project cost-benefit evaluations.)

- $\phi = 2$. Marginal cost of intervention is higher at high values of *q*, but not too extreme.

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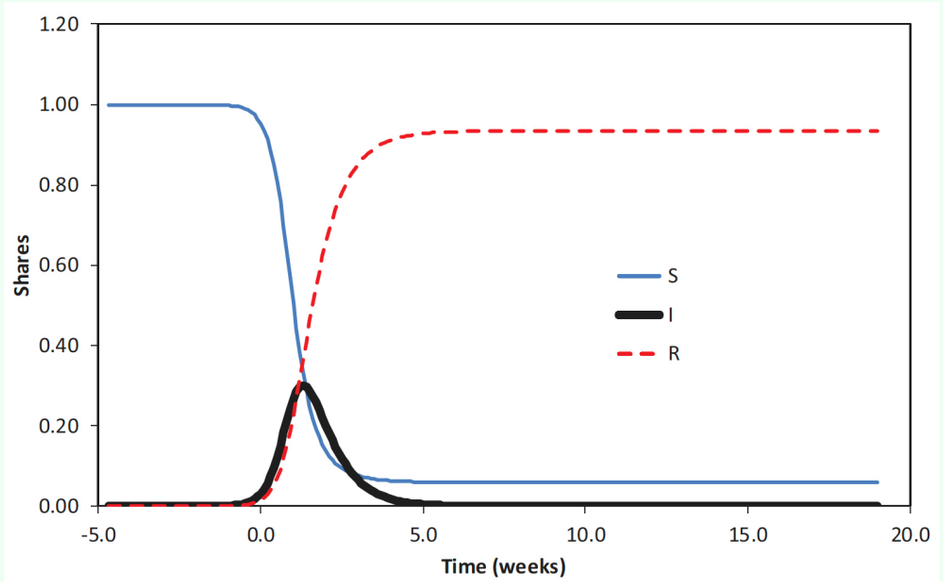
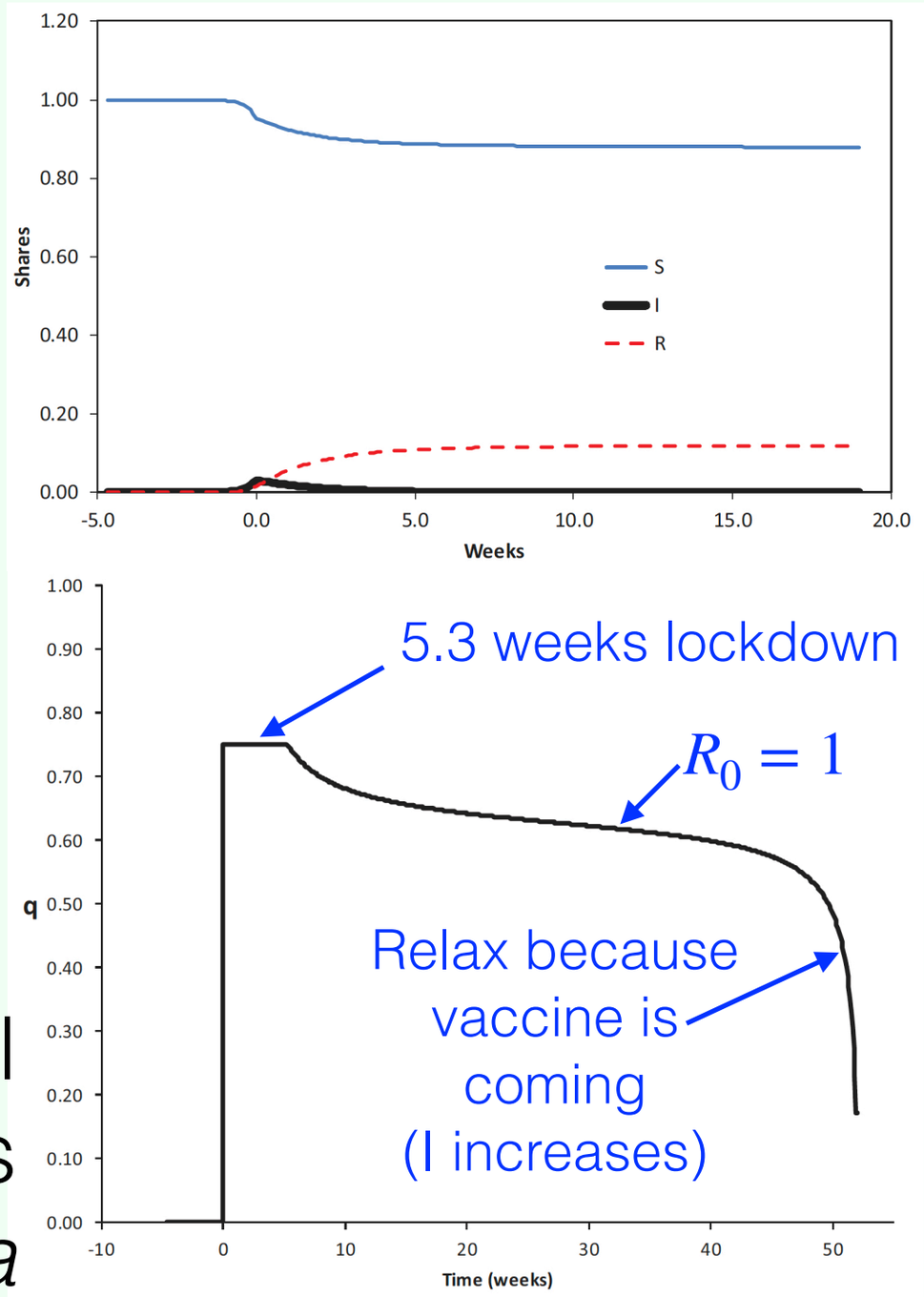
Some results



No intervention
440000 deaths
E = £14000 *per capita.*



Baseline optimal
60000 deaths
E = £6600 *per capita*

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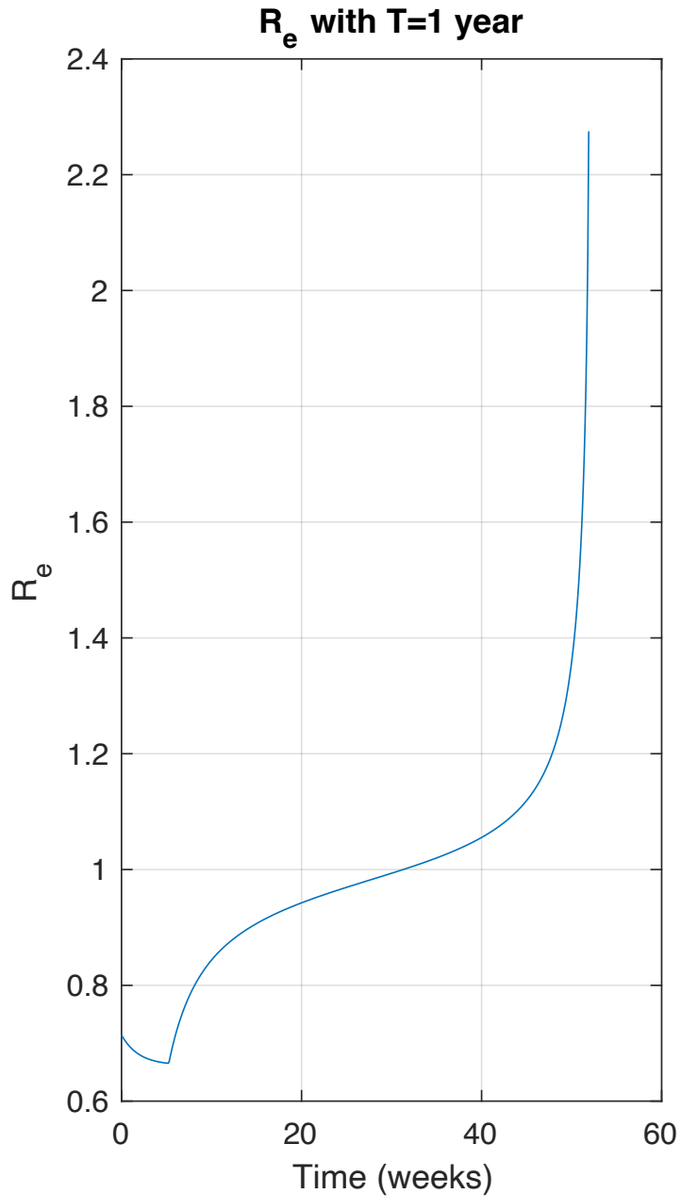
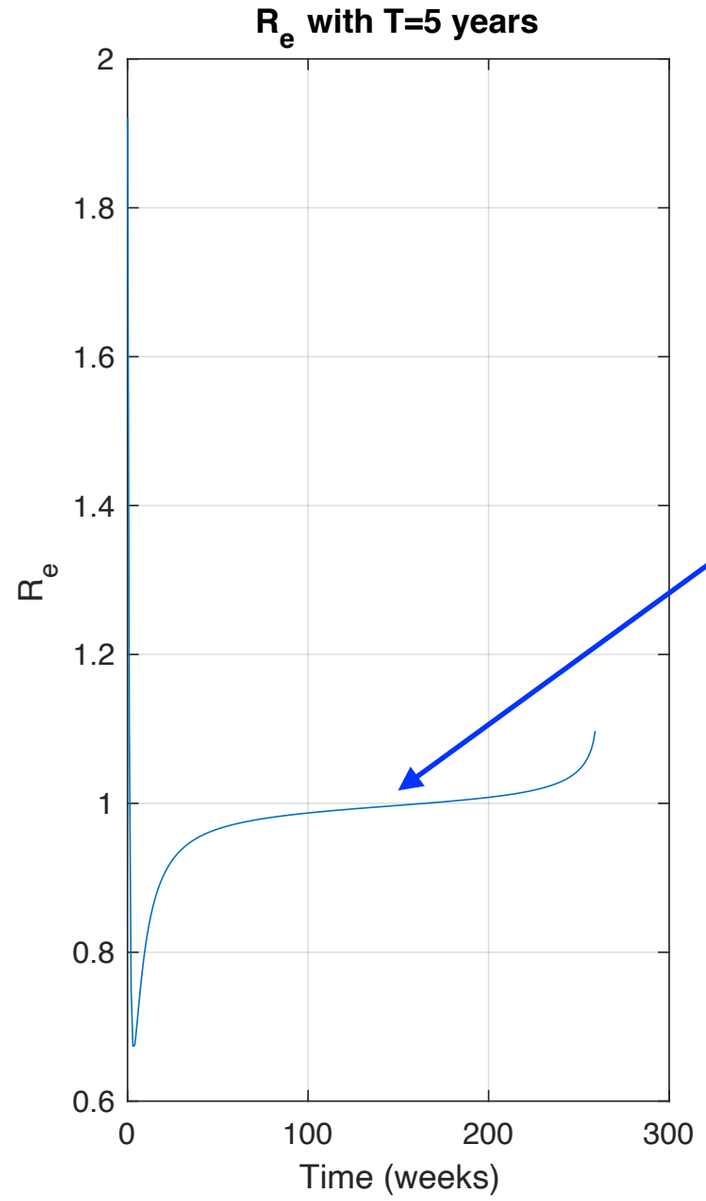
Effective reproduction number (baseline case)

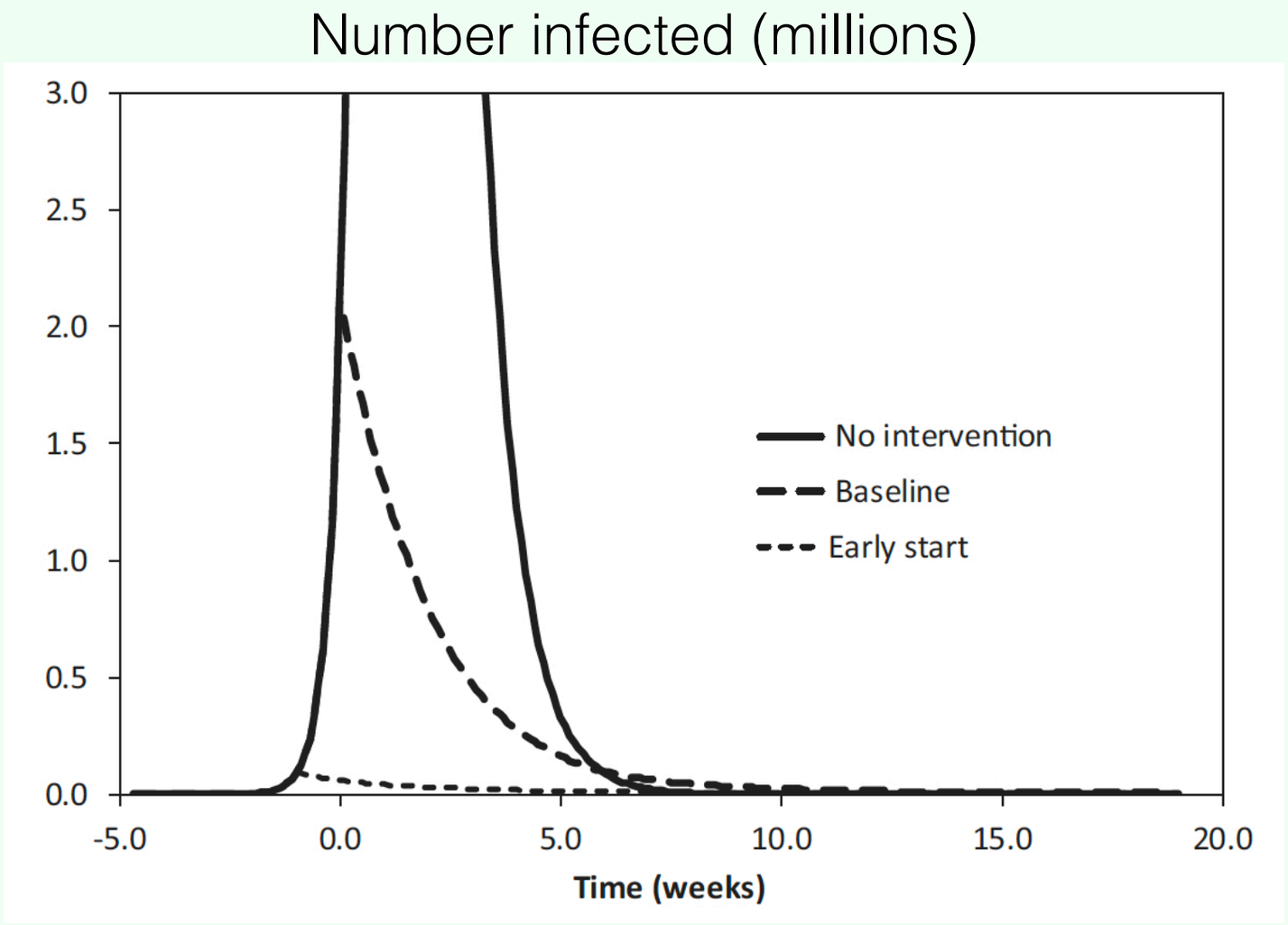




$R_e \approx 1$ for about 3 out of 5 years

Early start

- Start intervention 1 week earlier
- Lockdown: 0.9 weeks
- 8000 deaths
- *E*=£7400 *per capita*



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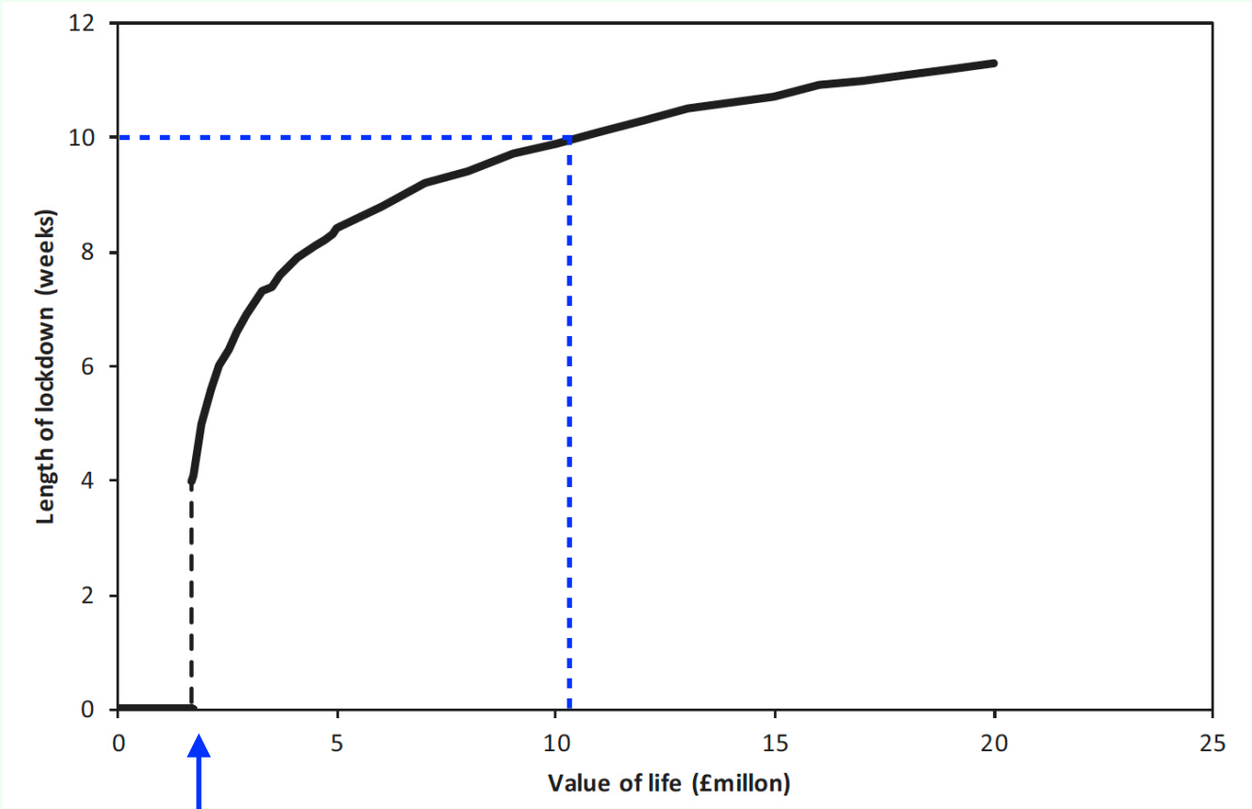
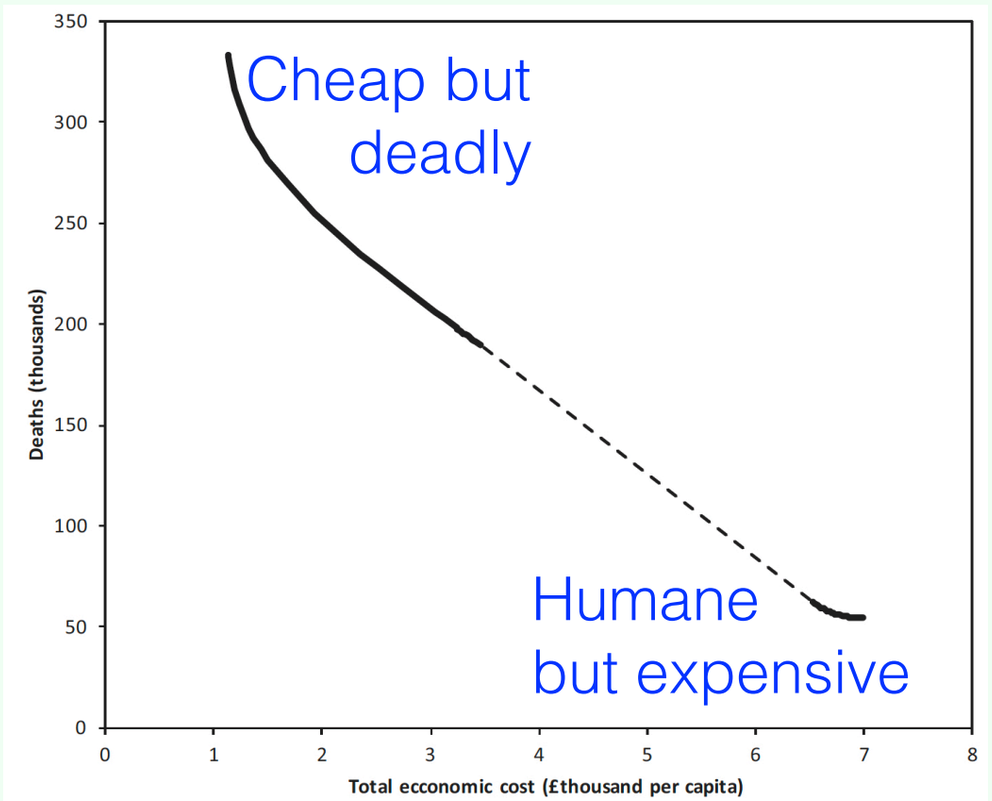
Discontinuous scenarios



$\pi_D \approx 1700$



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Effects of changing ϕ

$\pi_A = 2$, $\pi_D = 2000$

ϕ	Lockdown	
1	7.9 weeks	
2	5.3 weeks	← Baseline
4	1.8 weeks	

10-week lockdown requires
if $\phi = 2, \pi_A = 2$ then $\pi_D > 10000$,
if $\phi = 1, \pi_A = 3$ then $\pi_D > 4000$.

Much more than values usually used by UK government.

Value apparently used by NICE: $\pi_D = 300$

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π_D assumed by NICE

Age range	Deaths per 100k	Life expectancy	Value of life (£k)*
< 65	5	50.7	1522
65 - 69	45	18.4	552
70 - 74	87	14.6	437
75 - 79	174	11.1	333
80 - 84	387	8.1	243
85 - 90	720	5.7	171
> 90	1456	3.8	155

*Assumes £30k per year of life foregone.

Weighted average gives $\pi_D = 300$ per capita.

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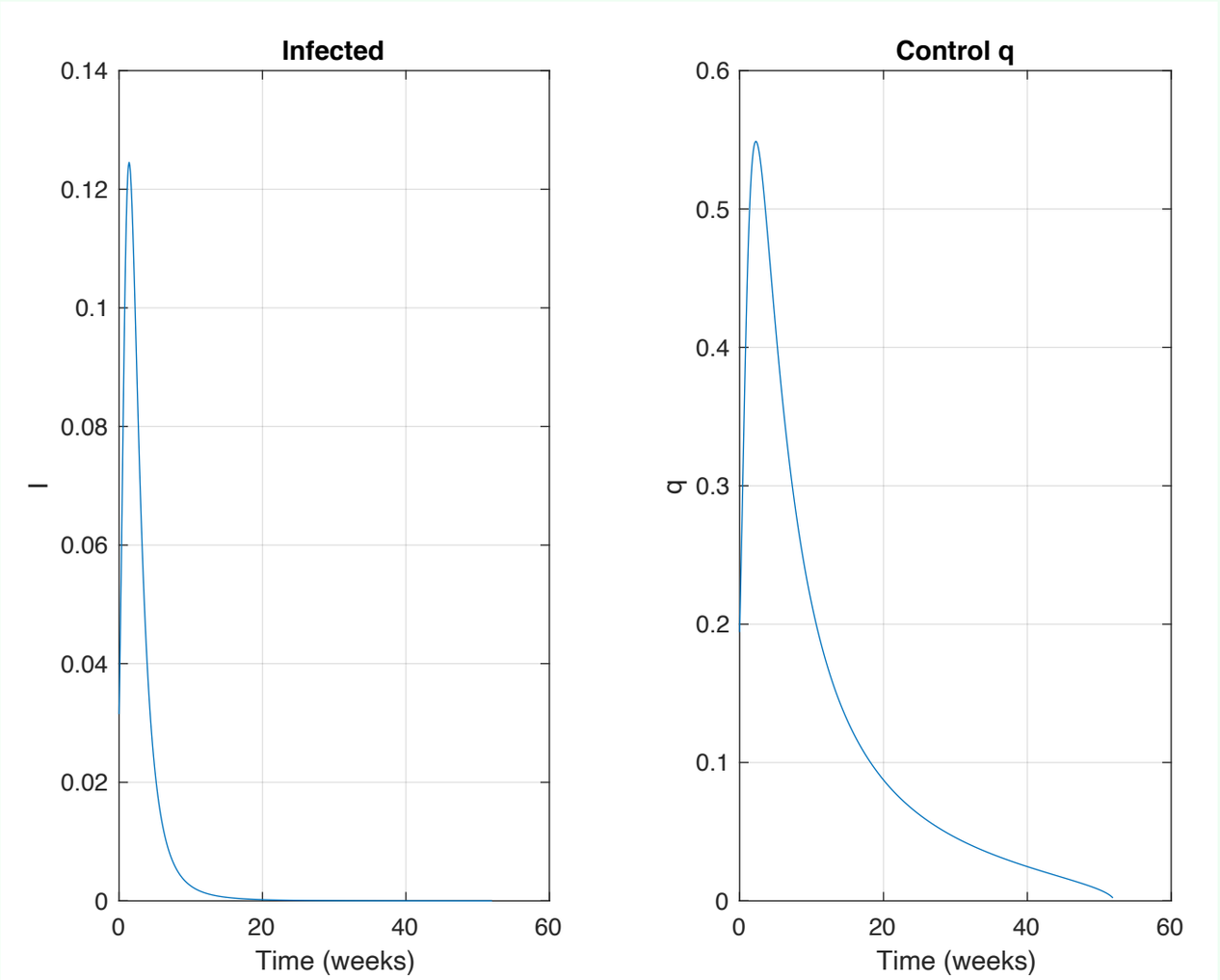
Optimal scenario with $\pi_D = 300$



Economic cost £1175 *per capita*, 317655 deaths.

Test and Trace

- Available for testing: aI (consider infected people only)
- Imperfect testing:
  Prob{Negative result in period $s\} = e^{-ps}$
- Prob{Positive result at some time}
  $= 1 - \int_0^\infty \gamma e^{-(p+\gamma)s} ds = \frac{p}{p+\gamma}$
- Testing capacity = M per week

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Test and Trace

- Fraction b of tests taken by infected people.
  Then $p = \frac{\gamma a I}{\gamma a I - bM}$ if capacity-constrained

- Imperfect tracing/isolation: For each positive result, c people are isolated — and considered Removed

- Test and Trace starts at $t = T^*$

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Test and Trace

Then

$$\frac{dI}{dt} = (1-q)\beta_0 SI - \gamma I - Q(t,I)$$

$$\frac{dR}{dt} = \gamma I + Q(t,I)$$

where $$Q(t,I) = \begin{cases} 0 \text{ if } t < T^* \\ c\min(bM, \gamma aI) \text{ if } t > T^* \end{cases}$$

Parameters: $a = 0.5, b = 0.5, c = 1.6,$
$M = 0.021$ (capacity is 1.4 million tests per week)

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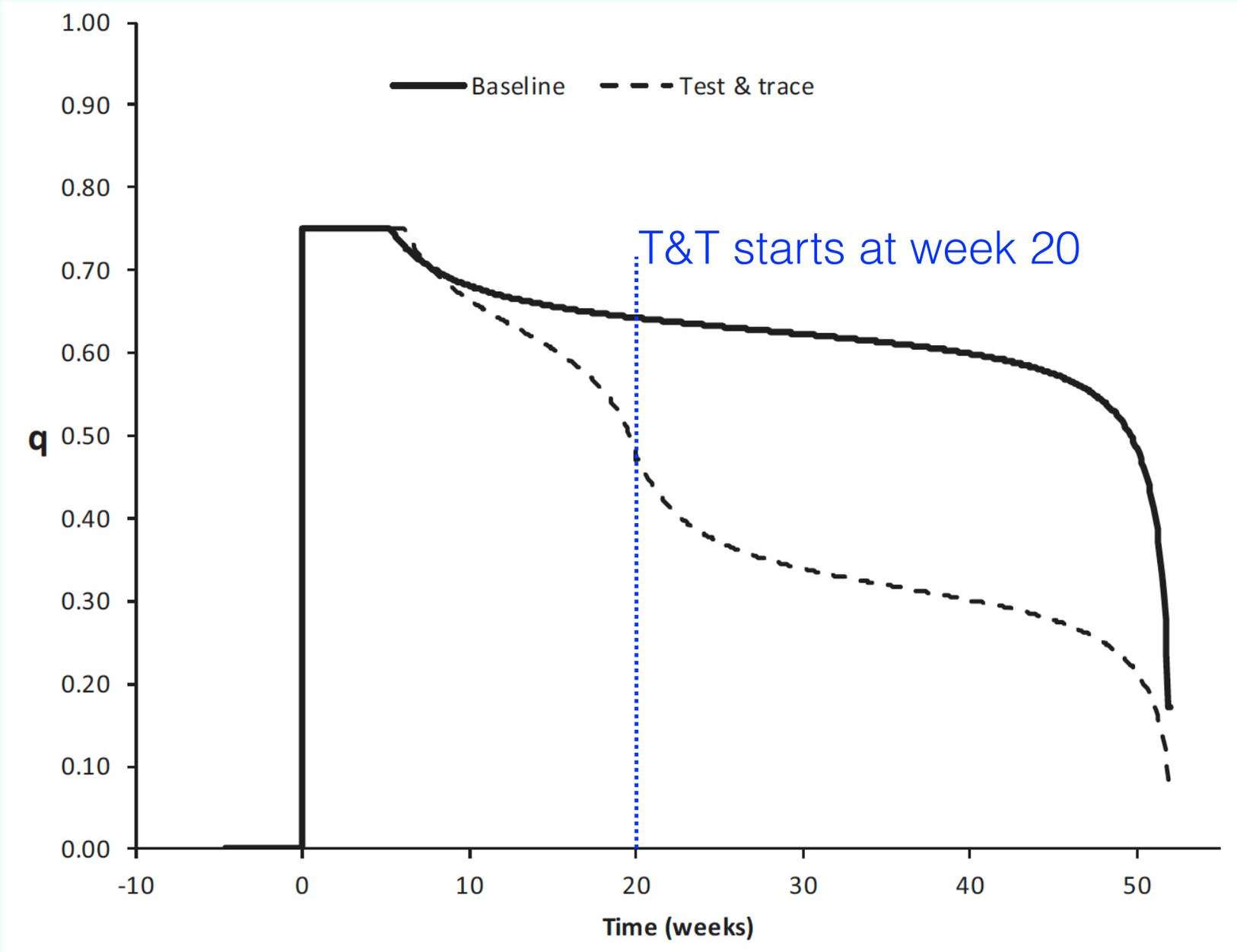
Test and Trace

Lockdown increased from 5.3 to 6.0 weeks.

Deaths remain at 60000

E reduced from £6600 to £3500



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Should our policy be implemented? NO!

- Open-loop, perfect foresight, no robustness.
- Relaxation at end disastrous if vaccine is late.
- OK to implement our policy in receding-horizon manner, ie Model Predictive Control, maybe event-driven not time-driven.
- Robustify, eg min-max-min *J* and/or stochastic.
- Use more fancy model, and re-estimate. δ likely to change.

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Conclusions

- We present a methodology, not a policy.
- Cost-benefit trade-off can be posed as an optimal control problem.
- Numerical optimisation is flexible, allows for ad-hoc constraints, eg Intensive Care capacity.
- UK lockdown is consistent with larger value of life than is normally used by government departments.